Lecture Notes, January 6, 2010, part 1

Partial and General Economic Equilibrium

PARTIAL EQUILIBRIUM

$S_k(p^o_k) = D_k(p^o_k)$, with $p^o_k > 0$ (or possibly, $p^o_k = 0$), or

$p^o_k = 0$ if $S_k(p^o_k) > D_k(p^o_k)$.

GENERAL EQUILIBRIUM For all i = 1,...,N,

$D_i(p^o_1, p^o_2,...,p^o_N) = S_i(p^o_1,..,p^o_N)$, $p^o_i > 0$, and

$p^o_i = 0$ for goods i such that

$D_i(p^o_1,...,p^o_N) < S_i(p^o_1,...,p^o_N)$.

What's wrong with partial equilibrium? Suppose there's no consistent choice of $(p^o_1,..,p^o_N)$. Then there would be (apparent) partial equilibrium --- viewing each market separately --- but no way to sustain it, because of cross-market interaction. Competitive equilibrium is supposed to make efficient use of resources by minimizing costs and allowing optimizing consumer choice. But how do we know prices in other markets reflect underlying scarcity assuming "other things being equal" . If not, then apparently efficient equilibrium allocation may be wasteful. A valid notion of equilibrium and efficiency needs to take cross-market interaction into account.

Three big ideas

Equilibrium: $S(p) = D(p)$
Decentralization
Efficiency

The Robinson Crusoe Model

q = oyster production
c = oyster consumption
168 (hours per week) endowment
L = labor demanded
R = leisure demanded
168-R =labor supplied

$q = F(L)$

$R = 168 - L$

Centralized Allocation
We assume second order conditions so that local maxima are global maxima:

$$F'' < 0, \frac{\partial^2 u}{\partial c^2} < 0, \frac{\partial^2 u}{\partial R^2} < 0.$$

u(c,R) = u(F(L), 168 - L)

$$\max_L \; u(F(L),\ 168 - L)$$

$$\frac{d}{dL} u(F(L),\ 168 - L) = 0$$

$$u_c F' - u_R = 0$$

$$\left[-\frac{dq}{dR}\right]_{u=u\max} = \frac{u_R}{u_c} = F'$$

Pareto efficient

$$MRS_{R,c} = MRT_{R,q} (= RPT_{R,q}\ )$$

Decentralized Allocation
$\Pi = F(L) - wL = q - wL$
Income:
$Y = w \cdot 168 + \Pi$
Budget constraint:

Y = wR + c

Equivalently, c = Y- wR = Π + wL = Π + w(168-R), a more conventional definition of a household budget constraint.

Firm profit maximization:
$\Pi = q - wL$

$$\frac{d\Pi}{dL} = F' - w = 0, \text{so } F'(L^0) = w$$

Household budget constraint:
$$wR + c = Y = \Pi^0 + w168$$

Choose c, R to maximize u(c, R) subject to (1.14). The Lagrangian is

V = u(c,R) - λ (Y - wR - c)

$$\frac{\partial V}{\partial c} = \frac{\partial u}{\partial c} + \lambda = 0$$

$$\frac{\partial V}{\partial R} = \frac{\partial u}{\partial R} + \lambda w = 0$$

Dividing through, we have

$$MRS_{R,c} = \left[-\frac{dc}{dR}\right]_{u=constant} = \frac{\frac{\partial u}{\partial R}}{\frac{\partial u}{\partial c}} = w$$

$$wR + c = w168 + \Pi^0$$

$$c = w(168 - R) + \Pi^0$$

Walras' Law

Note that the Walras Law holds at all wage rates --- both in and out of equilibrium. It is not an equilibrium condition.

$Y = w \cdot 168 + \Pi = w168 + q - wL \quad = wR + c$

$0 = w(R - (168 - L)) + (c - q)$

$0 = w(R + L - 168) + (c - q)$

Definition : Market equilibrium. Market equilibrium consists of a wage rate w^0 so that at $w^0, q = c$ and $L = 168 - R$, where q, L are determined by firm profit maximizing decisions and c, R are determined by household utility maximization. (in a centralized solution L=168-R by definition; in a market allocation wages and prices should adjust so that as an equilibrium condition L will be equated to 168-R).

Profit maximization at w^0 implies $w^0 = F'(L^0)$.
Utility maximization at w^o implies

$$\frac{u_R(c^0, R^0)}{u_c(c^0, R^0)} = w^0$$

Market-clearing implies $R^0 = 168 - L^0, c^0 = F(L^0)$.

So

$$F' = \frac{u_R}{u_c}$$

which implies Pareto efficiency.